Abstract

In a world with taxes, there is a small discrepancy between the deflated WACC \( \text{WACC}_{\text{Def}} \) and the real \( \text{wacc} \). This is due to the \((1-T)\) term that is in the standard expression for the WACC applied to the Free Cash Flow (FCF). We compare different approaches for valuing nominal and real cash flows with the 1) nominal Weighted Average Cost of Capital, WACC, 2) real WACC, \( \text{wacc} \), 3) inflated WACC, \( \text{WACC}_{\text{Inf}} \) and 4) deflated WACC, \( \text{WACC}_{\text{Def}} \). The cash flows are derived from financial statements that have been constructed in nominal prices. As a general conclusion or consistency in valuation, we must use the deflated WACC rather than the real WACC to discount real cash flows, and the nominal WACC to discount nominal cash flows.

Keywords: Weighted Average Cost of Capital, WACC, firm valuation, capital budgeting, deflated WACC, real WACC, inflation.

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Introduction

In a world without taxes, the value of the deflated Weighted Average Cost of Capital, WACC applied to the real Free Cash Flow (FCF) equals the value of the real WACC applied to the real FCF. However, in a world with taxes, there is a small discrepancy between the two WACCs. This is due to the (1-T) term that is in the standard expression for the WACC applied to the Free Cash Flow (FCF). Why is this important?

For consistency in valuation, we discount the nominal cash flows with the nominal discount rate, and the corresponding real cash flows with the corresponding real discount rate, and both of these calculations must give the same present value. If not, then we know that there is a mistake in the financial model and the valuation. Since there is a discrepancy between the deflated WACC, WACCDef, and the real WACC, wacc, in a world with taxes, using the wrong WACC leads to a mismatching of results that is due to the WACC rather than the financial model. Similarly, there is a difference between the inflated WACC, WACCInf, and the nominal WACC, WACC, as well.

The value of the discrepancy is small and the formulas for the discrepancies, as derived by Bradley and Jarrell, (2008), are as follows:

\[ \text{Difference in the real formulation} = \text{wacc} - \text{WACCDef} \]
\[ \text{Difference in the nominal formulation} = \text{WACCInf} - \text{WACC} \]

When estimating the nominal or real Kd or Ke we rely on a proxy similar to the Capital Asset Pricing Model, CAPM. CAPM states that a nominal interest rate (a rate of return) has the three above mentioned components: inflation, risk and real interest rate. The CAPM says

\[ R = R^n + \pi + R_i \]

Where \( R \) is a nominal return, \( R^n \) is the risk free rate, \( \pi \) stands for the return to inflation, and \( R_i \) stands for the return to the risk premium. 

We organize this note as follows. First, we explain the distinction between the two WACCs: WACCDef and wacc. Second, we provide a simple numerical example to illustrate the difference. Third, we conclude. In Appendix A, we provide the algebraic derivation of the difference.

Section One. Real, Nominal, Deflated and Inflated WACC

In the economics and financial literature, the real rate of interest is associated with the deflated rate of interest. We do recognize that a current interest rate has three components: inflation, risk and real interest rate. Hence, when we refer to the real rate we are assuming no risk premium and an inflation free rate. This real rate of interest is not observable in the economy, but it can be estimated by deflating the risk free rate, \( R \), with the expected inflation rate.

We use the Fisher equation in its exact multiplicative form. This is, "the rate of interest in the (relatively) depreciating standard is equal to the sum of three terms, viz., the rate of interest in the appreciating standard, the rate of appreciation itself, and the product of these two elements" (Fisher, 1896, 8-9; emphasis in original, cited by Dimand, 1999, p. 746). Fisher concluded that "The adjustment of (money) interest to long price-movements is more perfect than to short price-movements" (1907, 283; emphasis in original). Fisher, 1930, studied this relationship and examined its statistical importance with the correlation between lagging inflation and interest rates. This is admirable given the restricted computing resources available at that time.

\[ 1 + \text{RATE} = (1 + \text{rate})(1 + \pi) = 1 + \text{rate} + \pi + \pi \times \text{rate} \]  
\[ = 1 + \text{rate} + \pi \]  
\[ (1a) \]

Where \( \pi \) is the expected inflation rate. WACCDef, based on parameters in real terms, and WACC, which is obtained from WACC, based on parameters in nominal terms and the Fisher relationship. WACCInf, which is defined using the Fisher relationship, does not equal the wacc. As stated earlier, this discrepancy between the real and deflated WACCs is due to the coefficient (1-T) that is applied to the cost of debt in the expression for the WACC in equation (3a).

In this work we assume the real rate is constant across borders and time. This is not strictly true, but on average it tends to be constant. This is suggested explicitly or implicitly by McMillan, Buck, and Deegan, (1984), Woodward, (1992), Kennedy, (2000), Cremers, (2001) and Chung and Crowder, (2004) when they study the real interest rate parity.

In the case of WACC, there is a distinction between wacc, based on parameters in real terms, and WACCDef, based on parameters in nominal terms and the Fisher relationship. There are two possible interpretations of the expression for the WACC in equation (5a). First, in an unrealistic world where the expected inflation rate is zero, the cost of debt and the return to equity in equation 1 are in real terms.

\[ \text{Real WACC = wacc = D\% \times Kd(1 - T) + E\% \times Ke} \]  
\[ (3b) \]

Where D\% is the percentage of debt, T is the tax rate and p is the expected inflation rate.

From a conceptual point of view, the distinction is important, even though the differences in the valuation may be small.

Second, in a world with a positive expected inflation rate, the cost of debt and the return to equity in the expression for WACC (as in equation (3c)), and via the Fisher relationship, we obtain WACCInf in equation (4a). For WACCInf we use the same Fisher relationship using wacc multiplied by one plus the inflation rate, plus the inflation rate.

The standard relationships between the nominal and real values for the cost of debt and the return to equity, via the Fisher relationships, are as follows:

\[ 1 + Kd = 1 + kd(1 + \pi) = 1 + kd + \pi + \pi \times kd \]  
\[ = 1 + kd \]  
\[ (5.1) \]

\[ 1 + Ke = 1 + ke(1 + \pi) + \pi = (1 + ke) \times (1 + \pi) \]  
\[ = (1 + ke) \times (1 + \pi) \]  
\[ (5.2) \]

When estimating the nominal or real Kd or Ke we rely on a proxy similar to the Capital Asset Pricing Model, CAPM. CAPM states that a nominal interest rate (a rate of return) has the three above mentioned components: inflation, risk and real interest rate. The CAPM says

\[ R = R^n + \pi + R_i \]  
\[ (6a) \]

Where \( R \) is a nominal return, \( R^n \) is the risk free rate, \( \pi \) stands for the return to inflation, and \( R_i \) stands for the return to the risk premium.

These two previous equations are based on the Fisher relationship.

To obtain WACCInf, we subtract the expected inflation rate from the nominal WACC and divide by one plus the expected inflation rate. Using nominal values for the cost of debt and the return to equity in the expression for WACC (as in equation (3c)), and via the Fisher relationship, we obtain WACCInf in equation (4a). For WACCInf we use the same Fisher relationship using wacc multiplied by one plus the inflation rate, plus the inflation rate.

We call this wacc and distinguish this real WACC in equation (3b) from WACCDef, as defined below.

\[ 1 + Kd = 1 + kd + \pi \times kd = 1 + kd + \pi(1 + kd) \]  
\[ = 1 + kd + \pi + \pi \times kd \]  
\[ (3.1) \]

Second, in a world with a positive expected inflation rate, the cost of debt and the return to equity in equation (2a) are in nominal terms.

\[ \text{Nominal WACC = WACC = D\% \times Kd(1 - T) + E\% \times Ke} \]  
\[ (3c) \]

Where Kd is the real cost of debt and Ke is the real return to levered equity.


In the literature it is also common to consider a real interest rate as a deflated interest rate even if the nominal rate is the Rf or the return of an investment in the stock market. See for instance, Huizinga and Mishkin, (1984), Kandel, Ofer and Sarig, (1996) and Das, (2004). For us, the real interest rate comes from the deflated risk free rate; others are deflated rates.

In the context of cash flow valuation, the standard textbook formula for WACC applied to the FCF is as follows:

\[ \text{WACCDef} = D\% \times Kd(1 - T) + E\% \times Ke \]  
\[ (4a) \]

In the same vein we define WACCInf as

\[ \text{Inflated WACC} = \text{WACCInf} = \text{wacc} \times (1 + \pi) + \pi \]  
\[ (4b) \]
Joseph Tham, Ignacio Vélez-Pareja.

"real" return, r', with only inflation excluded. For the case of r', the CAPM formulation is

\[ r' = r + \beta \times (R_m - R_f)/ (1 + \pi) \]  

Where r is the real rate of interest, estimated by deflating R_m, r' is the "real" return including risk and β stands for what is known as the beta for the stock. In fact, if we deflate (6b) using the correct Fisher equation we will obtain (6a).

If one is not careful, one could easily assume (mistakenly) that the expected inflation rate should not affect the value of the WACC. However, as we show with a numerical example in Section Two, in the presence of taxes, there is an important distinction between WACC^Def and wacc. The problem lies with the (1-t) coefficient applied to the cost of debt in the expression for the WACC, and this intuition is correct.

Let us consider the different approaches:

\[ FCf_n = fcfn \times (1+\pi)^n \]  

Where FCf stands for the nominal free cash flow and fcfn for the real free cash flow.

Case 1. Nominal FCF discounted with the WACC

\[ PV (FCf @ WACC) = \sum_{n=0}^{N} \frac{FCf_n}{(1+WACC)^n} \]  

Case 2. Real FCF, fcfn, discounted with the wacc

\[ PV (fcfn @ wacc) = \sum_{n=0}^{N} \frac{fcfn}{(1+wacc)^n} \]  

Case 3. Real FCF, fcfn, discounted with the WACC^Def.

\[ PV (fcfn @ WACC^Def) = \sum_{n=0}^{N} \frac{fcfn}{(1+WACC^Def)^n} \]  

Case 4. Nominal FCF, discounted with the inflated WACC, WACC^Inf.

\[ PV (FCf @ WACC^Inf) = \sum_{n=0}^{N} \frac{FCf_n}{(1+WACC^Inf)^n} \]  

In the presence of taxes, wacc and WACC^Def are different. Hence, (8) and (10), and (9) and (11) are respectively identical. In a world without taxes the four previous expressions are identical.

**Section Two. A Simple Numerical Example**

In this section, we illustrate the distinction between wacc and WACC^Def and between WACC and WACC^Inf with a simple numerical example.

Consider the following numerical values.

- D% = 40%, E% = 60%, T = 20% and π = 5%, kd = 6% and Ke = 10%.

Calculation of the real WACC with parameters in real terms

\[
\begin{align*}
\text{wacc} &= D\% \times kd \times (1 – T) + E\% \times Ke \\
&= 40\% \times 6\% \times (1 – 20\%) + 60\% \times 10\% \\
&= 4.8\% + 6.00\% = 10.8\%
\end{align*}
\]

For calculating WACC we introduce Ke and Kd into the equation as follows:

Using 5.1

\[ \text{Kd} = 6\% \times (1 + 5\%) + 5\% = 11.300\% \]

Using 5.2

\[ \text{Ke} = 10\% \times (1 + 5\%) + 5\% = 15.500\% \]

Using (4)

\[ \text{WACC} = D\% \times Kd \times (1 - T) + E\% \times Ke \]

\[ = 40\% \times 11.3\% \times (1 - 20\%) + 60\% \times 15.5\% \]

\[ = 4.092\% + 9.300\% = 12.916\% \]

WACC^Def, WACC^Inf

\[ \text{WACC^Def} = \sum_{n=0}^{N} \frac{fcfn}{(1+WACC^Def)^n} \]

\[ \text{WACC^Inf} = \sum_{n=0}^{N} \frac{fcfn}{(1+WACC^Inf)^n} \]

The difference between wacc and WACC^Def is 0.381%. In a world without taxes the value of the real WACC, which is 7.92%, is US$ 1,016.11.

In Table 3, we show the present values of the two FCF (real and nominal) discounted with WACC, wacc, WACC^Def and WACC^Inf.

### Table 3. Present Value of Real and Nominal FCF at Different Inflation Rates

<table>
<thead>
<tr>
<th>Inflation rate</th>
<th>PV nominal CF at WACC</th>
<th>PV real CF at WACC</th>
<th>PV real CF at wacc</th>
<th>PV real CF at WACC^Def</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0%</td>
<td>1,016.11</td>
<td>1,016.11</td>
<td>1,016.11</td>
<td>1,016.11</td>
</tr>
<tr>
<td>2.5%</td>
<td>1,021.34</td>
<td>1,021.34</td>
<td>1,016.11</td>
<td>1,016.11</td>
</tr>
<tr>
<td>5.0%</td>
<td>1,026.36</td>
<td>1,026.36</td>
<td>1,016.11</td>
<td>1,016.11</td>
</tr>
<tr>
<td>7.5%</td>
<td>1,031.19</td>
<td>1,031.19</td>
<td>1,016.11</td>
<td>1,016.11</td>
</tr>
<tr>
<td>10.0%</td>
<td>1,035.83</td>
<td>1,035.83</td>
<td>1,016.11</td>
<td>1,016.11</td>
</tr>
<tr>
<td>12.5%</td>
<td>1,040.29</td>
<td>1,040.29</td>
<td>1,016.11</td>
<td>1,016.11</td>
</tr>
<tr>
<td>15.0%</td>
<td>1,044.59</td>
<td>1,044.59</td>
<td>1,016.11</td>
<td>1,016.11</td>
</tr>
</tbody>
</table>

Observe that the PV for nominal FCF at nominal WACC (Column 1) and real FCF at WACC^Def (Column 2) are identical and non neutral to inflation; also observe that they are consistent as they should be. The present value of the real FCF at wacc (Column 3) and the nominal cash flow at WACC^Def (Column 4) are identical as expected and are inflation neutral.

The table shows that inflation creates value and this may appear to be strange; the higher the expected inflation rate, the higher is the PV. However, this is consistent because the higher expected inflation rate means that the present value of the interest payments is higher and this in turn means that the present of the tax shields is higher. As can be seen in table 3, neither PV of real cash flows at wacc (Column 3) nor PV of nominal cash flows at WACC^Def (Column 4) are consistent with the PV of nominal cash flows at WACC (Column 2).

In the next table we show the same prevent values of the cash flows without taxes. In table 4 we show the present values of the two FCF (real and nominal) discounted with WACC, wacc, WACC^Def and WACC^Inf with no taxes.
As shown in Vélez-Pareja, (2006), under certain conditions, under the flow appraisal. The conditions, among others, include the existence of taxes, depreciation and accounts receivable. In this simple example we have assumed no depreciation and no accounts receivable, nor payable.

### Conclusion

In this note, using a simple numerical example, we have shown that in a world with taxes, there is a discrepancy between WACC and WACC, and between WACC and WACC. This means that under the restricted conditions of no depreciation, no accounts receivable and payable, it is equivalent and correct to value the nominal cash flows at WACC and the real cash flows at WACC. Correspondingly, it is wrong to value the real cash flows at WACC and the real cash flows at WACC. For consistency in valuation, we must use WACC rather than wacc in discounting real free cash flows, as proposed by Bradley and Jarrell (2008).

### Bibliographic References


### Appendix A

The expression for the nominal WACC is as follows:

\[ WACC = D\% \cdot kd \cdot (1 - T) + E\% \cdot ke \] (A1)

Substituting equations 5.1 and 5.2 into the right side of equation A1, we obtain,

\[ WACC = D\% \cdot kd \cdot (1 - T) + E\% \cdot ke - D\% \cdot \pi \cdot T/(1 + \pi) \] (A5.3)

Simplifying, we obtain,

\[ WACC = D\% \cdot kd \cdot (1 - T) + E\% \cdot ke + \pi \cdot D\% \cdot T/(1 + \pi) + \pi \cdot E\% \cdot T/(1 + \pi) \] (A4)

Rearranging, we obtain,

\[ WACC = D\% \cdot kd \cdot (1 - T) + E\% \cdot ke + \pi \cdot D\% \cdot T/(1 + \pi) \] (A5.3)

As we know, D% plus E% is 100%, then

\[ WACC = D\% \cdot kd \cdot (1 - T) + E\% \cdot ke + \pi \cdot D\% \cdot T/(1 + \pi) \] (A5.3)

Compare equation (3b) with equation A5.3. The extra term in A5.3 is the expression for the difference between wacc and WACC.

The other way around, if we begin with the wacc, and inflate it to WACC, we have:

\[ wacc = D\% \cdot kd \cdot (1 - T) + E\% \cdot ke \] (A6)

Replacing kd and ke from (5.1) and (5.2) in (A6) we have

\[ wacc = D\% \cdot kd \cdot (1 - T) + E\% \cdot ke + \pi \cdot D\% \cdot T/(1 + \pi) \] (A1a)

\[ wacc = D\% \cdot kd \cdot (1 - T) + E\% \cdot ke - D\% \cdot \pi \cdot T/(1 + \pi) \] (A5.3)

\[ wacc = D\% \cdot kd \cdot (1 - T) + E\% \cdot ke + \pi \cdot D\% \cdot T/(1 + \pi) \] (A4)

Hence, there is a difference between WACC and WACC.