Consistent value estimates from the Discounted cash flow (DCF) and residual Income (RI) models in M & M worlds without and with taxes[1]

Estimaciones consistentes del valor en los modelos de flujo de caja descontado (DCF) e ingreso neto residual en un mundo M & M con y sin impuestos

Estimativas consistentes do valor nos modelos de fluxo de caixa descontado (FCD) e lucro líquido residual em um mundo M & M com e sem impostos

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Abstract

Recently, Lundholm and O'Keefe (2000) identified the estimation of the WACC as an important reason for the discrepancy between the value estimates obtained from the Discounted Cash Flow (DCF) and Residual Income (RI) models. In this paper, I discuss how we can obtain consistent value estimates from the two models in M & M worlds without and with taxes.

It is common to assume that the return to unlevered equity is constant. Additionally, in practice, one assumes that the return to levered equity is constant although the debt-equity ratio is changing. In M & M worlds without and with taxes, it would be **inconsistent** to assume that the returns to unlevered and levered equity are constant **and** the debt-equity ratio is changing.

JEL codes. D61: Cost-Benefit Analysis - G31: Capital Budgeting - H43: Project evaluation

Key words: Cost of Capital, Weighted Average Cost of Capital (WACC), Cash Flow to Equity, Residual Income Model, Discounted Cash Flow Model, Valuation, Return to Equity

Resumen

Lundholm y O'Keefe (2000) identificaron a la estimación del CPPC (costo promedio ponderado de capital) como una importante razón para la discrepancia entre el valor obtenido a partir de las estimaciones del flujo de caja descontado (DCF) y ingreso neto residual. En este trabajo se discute cómo podemos obtener estimaciones consistentes de valor de los dos modelos en un mundo M & M (Modigliani y Miller) con y sin impuestos.

Es común suponer que el costo del patrimonio sin deuda es constante. Además, en la práctica, se supone que el costo del patrimonio con deuda es constante, aunque la relación deuda-capital esté cambiando. En el mundo M & M con y sin impuestos, no sería coherente suponer

Resumo

Lundholm e O'Keefe (2000) identificaram a estimativa do CMPC (custo médio ponderado de capital) como uma importante razão para a discrepância entre o valor obtido a partir das estimativas do fluxo de caixa descontado (FCD) e o lucro líquido residual. Neste trabalho se discute como podemos obter estimativas consistentes de valor dos dois modelos em um mundo M & M (Modigliani e Miller) com e sem impostos.

É comum supor que o custo do patrimônio sem dívida é constante. Além disso, na prática, supõe-se que o custo do patrimônio com dívida é constante, embora a relação dívida-capital esteja mudando. No mundo M & M com e sem impostos, não seria coerente supor

Entregado en 16/02/2012 | Aprobado en 18/04/2012.

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Palabras clave: análisis beneficio costo, presupuestación de capital, evaluación de proyectos, valoración de flujos de caja.

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Palavras-chave: análise custo-benefício, orçamentação de capital, avaliação de projetos, valoração de fluxos de caixa.

Introduction

Recently, Lundholm and O'Keefe (2000) identified the estimation of the Weighted Average Cost of Capital (WACC) as an important reason for the discrepancy between the value estimates obtained from the Discounted Cash Flow (DCF) Model and the Residual Income (RI) Model. To reconcile properly the differences in the results obtained from the two models in M & M worlds without and with taxes, the estimation of the WACCs and the returns to levered equity must be consistent. In the case of the Residual Income Model, the estimation of the annual returns to levered equity is particularly crucial because the annual returns to levered equity are the key components in the calculation of the annual residual incomes.[1]

Unfortunately, in the practical applications of the Discounted Cash Flow and the Residual Income Models, it is unclear whether the valuation is conducted in an M & M world. In practice, it is common for valuation analysts to assume that e, the return to levered equity, is constant even when the debt-equity ratio is changing. If we assume that ρ, the return to unlevered equity, is constant, then in an M & M world, it is inconsistent to assume a constant return to levered equity with a changing debt-equity ratio.[2] In the Residual Income Model, the cash flows to the equity holders are directly valued and thus it is extremely important to be explicit about the calculation of the return to levered equity as a function of the debt-equity ratio. Rather than identifying the problem with the WACC, one should simply go directly to the source of the problem, namely the estimation of the return to levered equity. After all, the calculation of the WACC is based on the return to levered equity. If the return to levered equity is properly estimated, the WACC will also be correct.

In an M & M world without taxes, the value of the WACC in year n **must be** equal to the value of the return to unlevered equity in year n. If one assumes that e, the return to levered equity, is constant and the debt-equity ratio is changing, then the WACC must change. Alternatively, if one assumes that ρ the return to unlevered equity is constant and the debt-equity ratio is changing, then e, the return to levered equity, must change.

Consistent Valuation in an M & M world

It is well known that in an M & M world without taxes, the present value in year n of the levered cash flows is equal to the present value in year n of the unlevered cash flows.[3] In addition, the value of the levered cash flows is equal to the sum of the present value of the cash flows to the debt holders and the cash flows to the equity holders.[4] Thus, for each year n, the present value of the cash flow to the equity holders is equal to the present value of the free cash flow minus the present value of the cash flow to the debt holders. In symbols,

$$En = V^L_n - D_n \tag{1}$$

1. The Residual Income in year n is defined as the difference between the Net Income (NI) in year n and the cost of equity in year n, based on the value of the equity at the beginning of year n.

$$RI_n = NI_n - e_n * E_{n-1}$$

where e_n is the required return to levered equity in year n. See equation 8, pg 12 in Lundholm and O'Keefe (2000).

2. Usually, the assumption about the return to unlevered equity is never explicitly stated.

3. In symbols, $V^L_n = V^{UL}_n = V^{FCF}_n$

where V^L_n is the present value of the levered cash flows and V^{UL}_n is the value of the unlevered cash flows or the free cash flow (FCF).

4. In symbols, $V^L_n = E_n + D_n$

where En is the present value in year n of the equity cash flow, VLn is the present value in year n of the levered cash flow, and Dn is the present value in year n of the cash flow to the debt holders. This relationship for valuing the cash flow to the equity holder is only correct in an M & M world, that is, a world where all the assumptions underlying the M & M theorems are assumed to be true. From a practical point of view, it may be reasonable to initiate the valuation exercise within the context of an M & M world. Later, we can examine the impact of deviations from the assumptions of an M & M world on the outcomes of the valuation exercise. For example, in developing countries with shallow capital markets, it may not be reasonable to assume an M & M world. However, in this case, we will have to specify clearly how the deviations from the M & M world affect the expressions for the calculations of the return to equity and the WACC.

Based on the equality in line 1, we can derive relationships for calculating the WACC in year n and the return to levered equity in year n.[5]

In this paper, I compare the magnitudes of the errors introduced into equity valuation with the RI model due to the violations of the assumptions that are required to hold in M & M worlds with and without taxes. Specifically, what are the magnitudes of the errors in valuation, if we assume that the returns to unlevered equity and levered equity are constant? In addition, I will examine how the violations affect the value estimates from the DCF model relative to the value estimates obtained from the RI model.[6]

In Section One, I will assume that the analysis is conducted in an M & M world with no taxes. That is, the value of the levered cash flows will be equal to the value of the unlevered cash flows. I will compare the value estimates in two cases. In the first case, I will assume that ρ, the return to unlevered equity is constant; in the second case, I will assume that e, the return to levered equity, is constant.

In Section Two, I will include the impact of taxes in the analysis. I will assume that the tax rate is 30%.[7] Here I will assume that e, the return to levered equity, is constant and the value estimates obtained with the Residual Income model are correct. I will compare the values obtained from the DCF model relative to the RI model, as a function of the different assumptions used in the estimation of the appropriate WACC. The inclusion of taxes means that the expression for the after-tax WACC will change as a function of the percentage of debt.

Section one

In this section, I will present the analysis for an M & M world without taxes and discuss the results. The detailed calculations are presented in Appendix A and Appendix B. The main result of this section is straightforward. In an M & M world without taxes and a constant ρ, the WACC remains constant. It is common to assume that ρ, the return to unlevered equity is constant.[8]

Consider a simple cash flow profile over two years, with \$600 at the end of year 1 and \$3,600 at the end of year 2. The data are summarized in the following table.

	Year	0	1	2
Free Cash Flow (FCF)			600.0	3,600.0
Cash Flow to debt holders (CFD)			600.0	600.0
Cash Flow to equity holders (CFE)			0.0	3,000.0

Table 1: Free Cash Flow, Cash Flow to debt holders and Cash Flow to equity holders

Thus, at the end of year 1, the equity holders will receive nothing and at the end of year 2, the equity holders will receive \$3,000. All of the cash flow in year 1 is used for payment to the debt holders. The after-tax cost of debt is 6%.[9] The key assumption is that the

5. The WACC (pre-tax) in year n is a weighted average of the cost of debt in year n and the return to levered equity in year n, where the weights are based on the percentages of debt and equity at the beginning of year n. $WACC_n = \%D_{n-1}*d + \%E_{n-1}*e_n$

 The WACC (after-tax) is $WACC_n = \%D_{n-1}*d*(1 - t) + \%E_{n-1}*e_n$

 and is applied to the original free cash flow (FCF), excluding annual tax savings from the tax shields.

 In an M & M world without taxes, the expression for the return to levered equity is as follows:

$$e_n = \rho + (\rho - d)*\frac{D_{n-1}}{E_{n-1}}$$

 where ρ is the return to unlevered equity (assumed to be constant), en is the return to levered equity in year n, En-1 is the value of the levered equity at the beginning of year n, and Dn-1 is the value of the levered equity at the beginning of year n. The expression for the return to equity implies that en, the return to levered equity in year n is a positive function of the debt-equity ratio. In an M & M world with taxes, the expression for the return to equity remains unchanged if r, the return to unlevered equity is used for discounting the tax shield.

6. As a basis for analysis and discussion, I will use the numerical example presented in the Appendix of the paper by Lundholm and O'Keefe (2000).

7. Once we introduce taxes, the question about the correct discount rate for the tax shield arises. To be specific, I will assume that the correct discount rate for the tax shield is r, the return to unlevered equity rather than d, the cost of debt. See Tham (1999) for arguments why it may be appropriate to use r as the discount rate for the tax shield.

8. Alternatively, one could assume that the return to levered equity is constant. In this case, if the debt-equity ratio changes, then the return to unlevered equity and the WACC must also change.

9. With this value for the cost of debt, the value of the debt at the end of year 1 is \$566 and the value of the debt at the end of year 0 is \$1,100. See line A1.1 and line A1.2 in Appendix A. The after-tax cost of debt is, d*(1 - t) is 6% where d is the cost of debt and t is the corporate tax rate. In this case, there is no tax so the cost of debt before and after tax are the same.

return to levered equity is exogenously determined and is equal to 20%.

There are several possibilities for valuation.

Case 1: Return to levered equity is constant

The return to levered equity is constant for year 1 and year 2. Since the debt-equity ratio changes from year 1 to year 2, this assumption implies that the return to unlevered equity is changing.[10]

Case 2: Return to unlevered equity is constant

The return to levered equity of 20% holds for year 2. Based on this, we determine the corresponding return to unlevered equity in year 2, and then assume that the return to unlevered equity is constant. If the return to unlevered equity is constant, it means that the return to levered equity in year 1 must change and we must estimate the return to levered equity in year 1.

The important point is that we must be specific about the parameter that is constant. Since the debt-equity ratio changes from year 1 to year 2, in an M & M world without taxes, we must be clear whether the return to levered equity is constant or the return to unlevered equity is constant. If we wish to conduct consistent valuation in an M & M world without taxes, and we assume that e, the return to levered equity is constant, then the return to unlevered equity must vary; conversely, if we assume that ρ, the return to unlevered equity is constant, then the return to levered equity must vary.

Return to unlevered equity in year 2

If we assume that the return to levered equity in year 2 is 20%, we can determine the value of ρ_2, the return to unlevered equity in year 2. That is,

$$e2 = \rho 2 + (\rho 2 - d)*\frac{D_1}{E_1} = 20\% \quad (2)$$

Based on line 2, the value of ρ_2 is 17.42%. See line A7 in Appendix A.

Value of levered equity at the end of year 0 and year 1

Using the return to levered equity in year 2 of 20%, we can determine the value of the levered equity at the end of year 1. See line A2 in Appendix A. The calculation of the value of the levered equity at the end of year 0 (or the beginning of year 1) depends on our assumption. Specifically, it depends on whether we assume that the return to levered equity or the return to unlevered equity is constant for year 1.

First, we can use the **same** value of 20% as the return to levered equity in year 1, even though the debt-equity ratio at the end of year 0 is different from the debt-equity ratio at the end of year 1. This calculation is clearly inconsistent in an M & M world with the assumption that ρ, the return to unlevered equity is constant.[11]

Alternatively, to be consistent with an M & M world, we can assume that the return to unlevered equity is constant and use the value of ρ in year 2 to calculate the value of the unlevered equity at the end of year 0 and thus, indirectly determine the value of the levered equity at the end of year 0. The alternative method will result in a different value for the levered equity at the end of year 1, and consequently, the debt-equity ratio will change, which in turn will affect the calculation of the WACC. In an M & M world without taxes, both the returns to unlevered and levered equity cannot be constant for both years.

The results from both of the calculations are shown in the tables below.

	Year	0	1	2
Value of debt		1,100.0	566.0	
Value of equity (levered)		2,083.3	2,500.0	

10. In addition to these two possibilities, we could also assume that the return to levered equity of 20% applies to the debt-equity ratio prevailing at the beginning of year 1. In this case, we would determine r1 the return to unlevered equity in year 1. Again, we have two choices for year 2. We could either assume that e_2 the return to levered equity in year 2 is equal to e1 the return to levered equity in year 1, or r_2 the return to unlevered equity in year 2 is equal to r1 the return to unlevered equity in year 1.

    The fact that the equity holder receives no cash flow in year 1 does not mean that the return to equity in year 2 can be used for year 1. The return to equity for year 1 depends on the debt-equity ratio at the beginning of year 1 and this is different from the debt-equity ratio at the beginning of year 2.

11. We can make this assumption consistent with an M & M world if we assume that r the return to unlevered equity changes from year 1 to year 2. See line B3.3 in appendix B for the calculation of the return to unlevered equity if the return to levered equity is assumed to be constant. The return to the unlevered equity is equal to the WACC.

	Year	0	1	2
Total Value = debt + equity (levered)		3,183.4	3,066.0	

Table 2a: Value of levered equity, with e_1 = 20%

If we use a value of 20% for the return to levered equity in year 1, the value of the levered equity at the end of year 0 is $2,083.3. See Table 2a and line B1 in Appendix B.

Alternatively, using ρ equal to 17.42%, we find that the value of the unlevered equity at the end of year 0 is $3,122.3, and from this we can deduce that the value of the levered equity at the end of year 0 is $2,022.2. See line A11 in Appendix A. Using the value of the levered equity at the end of year 0 and taking into account the debt-equity ratio at the end of year 0, we find that e_1, the return to levered equity for year 1, is 23.63%.

	Year	0	1	2
Value of debt		1,100.0	566.0	
Value unlevered equity		3,122.3	3,066.0	
Value levered equity		2,022.2	2,500.0	

Table 2b: Value of levered equity, with e_1 = 23.63%

In Table 2b, the value of the levered equity at the end of year 0 is based on valuation in an M & M world with a constant return to unlevered equity. The percentage difference between the two values of levered equity at the end of year 0 is approximately three percent.

$$\textbf{Percent Difference} = \frac{\mathbf{2{,}083.3 - 2{,}022.2}}{\mathbf{2{,}022.2}} = \mathbf{3.02\%} \qquad \mathbf{(3)}$$

The different values for the levered equity implies that the debt-equity ratios will be different at the end of year 0. The debt-equity ratios corresponding to the two cases in Table 2a and Table 2b are shown in Table 3a and 3b, respectively.

	Average	0	1	2
Percent Debt	26.51%		34.56%	18.46%
Percent Equity	73.49%		65.44%	81.54%
Debt-Equity Ratio	0.361		0.528	0.226

Table 3a: Debt-equity ratios with constant return to levered equity equal to 20%

Based on the different values for the return to equity in year 1, the values for the WACC in year 1 will be different. Using the values in Table 3a, we find that the WACC in year 2 is 17.42%, and the WACC in year 1 is 15.16%. See lineB3.3 in Appendix B. Using the values in Table 3b, we find that the WACC in year 1 is 17.42%, which is the same as the value of the WACC in year 2. See lineA12.3 and lineA13.3 in Appendix A.

	Year	0	1	2
Percent Debt	26.85%		35.23%	18.46%
Percent Equity	73.15%		64.77%	81.54%
Debt-Equity Ratio	0.367		0.544	0.226

Table 3b: Debt-equity ratios with constant return to unlevered equity equal to 17.42%.

As expected, in an M & M world without taxes, the value of the WACC does not change. Although the debt-equity ratio changes, with the assumption of a constant return to unlevered equity, the return to levered equity adjusts to maintain a constant WACC.

In addition to the two values of the WACC presented here, there is a third possibility for the WACC. If one were to find the **single** WACC of the FCF with the assumption of constant return to levered equity equal to 20%, we see that the value is 16.18%. See line B4.2 in Appendix B. This value for the WACC has no clear meaning or interpretation in the context of valuation.[12] It is a kind of "weighted average" of the WACC in year 1 and the WACC in year 2.

At the end of year 0, what is the correct present value of the cash flows to the equity holder (levered)? Is it $2,083.3 or $2,022.2? The answer will depend on the parameter that is held constant. If the return to levered equity is held constant, and the return to unlevered

12. Technically, it is the IRR of the cash flow. But from a valuation point of view, what is it? Lundholm and O'Keefe (2000) cannot use this numerical example to illustrate their criticism about the use of the incorrect WACC value of 16.18%, if they assume that the return to unlevered equity is constant. A WACC calculated in violation of the conditions appropriate in an M & M world cannot be used to illustrate their point. They will have to use an example in an M & M world with taxes.

equity is allowed to change, then the correct value is \$2,083.3. Alternatively, if the return to unlevered equity is held constant, and the return to levered equity is allowed to change, then the correct value is \$2,022.2.

If we violate the assumptions of an M & M world, and use a constant value for the return to levered equity (at the same time, assuming a constant return to unlevered equity), the calculations will **overstates** the value of the levered equity at the end of year 0 by 3 percent, relative to the correct value under the assumption that the return to unlevered equity is constant.

Return to levered Equity in year 1	Levered Equity	% Debt	D/E Ratio	WACC
20.0%	2,083.3	34.6%	0.528	15.16%
20.5%	2,074.7	34.6%	0.530	15.48%
21.0%	2,066.1	34.7%	0.532	15.79%
21.5%	2,057.6	34.8%	0.535	16.10%
22.0%	2,049.2	34.9%	0.537	16.41%
22.5%	2,040.8	35.0%	0.539	16.72%
23.0%	2,032.5	35.1%	0.541	17.03%
23.5%	2,024.3	35.2%	0.543	17.34%
24.0%	2,016.1	35.3%	0.546	17.65%
24.5%	2,008.0	35.4%	0.548	17.95%
25.0%	2,000.0	35.5%	0.550	18.26%

Table 4: Relationship between the return to levered equity in year 1 and the percentage debt, the debt-equity ratio and the WACC in year 1.

In light of this discussion, we are in a position to examine the magnitude of the errors involved with the different assumptions. We will assume that ρ, the return to unlevered equity is constant, and estimate the magnitude of the error in assuming that the return to levered equity is also constant. If we assume that ρ, the return to unlevered equity is constant, then the correct value is \$2,022.2. At the same time, if we assume **inconsistently** that e, the return to levered equity is also constant at 20%, then the magnitude of the error will be the difference between the correct value of \$2,022.2 and the incorrect value of \$2,083.3.

Table 4 shows the relationship between the return to levered equity in year 1 and various parameters, namely, the value of the levered equity at the end of year 0, the percentage of debt at the end of year 0, the debt-equity ratio at the end of year 0 and the WACC for year 1. If we use a return to levered equity for year 1 different from 20%, then the magnitude of the errors will be the difference between the values in Table 4 corresponding to the values of the levered equity and correct value of \$2,022.2.

Discrepancies in the value estimates from the Discounted Cash Flow Model

Suppose the correct value, based on a constant return to unlevered equity of 17.41%, is \$2,022.3 An analyst, using a constant return to levered equity of 20% for year 1 and year 2 will obtain a value of \$2,083.3 with the Residual Income model. Another analyst, using a constant WACC of 16.18% will also obtain a value of \$2,083.3.4 with the Discounted Cash Flow model. Both of the value estimates from the two models are incorrect, relative to the correct value of \$2,022.2, but there is **no discrepancy** in the value estimates from the DCF and RI models. Moreover, a third analyst, using a WACC of 15.16% for year 1 and 17.42% for year 2 will also obtain the same value. Again, the value estimate is incorrect, but relative to the correct value of \$2,022.2 there is **no discrepancy** in the value estimates from third approach and the previous two estimates from the DCF and RI models. In summary, the value estimates of all three analysts from the Discounted Cash Flow Model and the Residual Income Models will be identical and incorrect.

There are other possible ways to calculate the WACC. For example, we could use the average debt percent over the two years, the debt percent in the first year or the debt percent in the second year. The following table summarizes the values for the cash flow to the levered equity holders for the different values of the WACC.

WACC values	Levered Value	Levered Equity	Difference
16.18%	3,183.37	2,083.33	61.09
16.24%	3,180.52	2,080.49	58.24
15.07%	3,240.23	2,140.19	117.95
17.42%	3,122.06	2,022.02	-0.23

Table 5: The values of levered equity based on different WACC values.

If the average percent debt over the two years is used in the calculation of the WACC, the value of the WACC is 16.24% and the levered equity is \$2,080.5. If the percent debt in the first year is used in the calculation of the WACC, the value of the WACC is 15.07% and the levered equity is \$2,140.2 and if the percent debt in the second year is used in the calculation of the WACC, the value of the WACC is 17.42% and the levered equity is \$2,022. The fourth column in Table 5 shows the difference between the values of the levered equity and the assumed correct value of \$2,022.3. The magnitude of the errors in the value estimates from the DCF model will depend on the assumptions used in the estimation of the WACC.

Section two

In this section, we examine the impact of taxes on the calculation of the return to levered equity and the WACC in year 1 and year 2. We will assume that the tax rate is 30%. The existence of taxes will require the estimation of the present value of the tax shield at the end of year 0 and year 1. The detailed calculations are presented in Appendix C. We will continue to assume that the after-tax cost of debt is 6%, which means that d, the cost of debt is 8.57%. Using this value for the cost of debt, we find that the value of the debt at the end of year 1 is $552.6, and the value of the debt at the end of year 0 is 1,061.6. See line C1.1 and line C1.2 in Appendix C.

Next, we have to determine the interest payments and the value of the tax savings.[13] In year 2, the tax savings are $14.21, and in year 1, the tax savings are $27.29. See line C5.1 and line C5.2 in Appendix C. The present value of the tax savings at the end of year 0 and year 1 will depend on the returns to unlevered equity in year 0 and year 1. To calculate the present value of the tax savings, we have to calculate the return to unlevered equity.

Here we assume that in year 2, the return to levered equity is constant at 20%. With this assumption, we can estimate the value of the cash flows to the equity holders (levered). See line C2.1 and line C2.2 in Appendix C. With these values, we can determine the percent of debt and equity, and calculate the WACC for year 1 and year 2. The returns to unlevered equity in year 1 and year 2 are equal to the WACC in year 1 and year 2, respectively.

13. The loan schedule is shown below.

Loan Schedule

	Year	0	1	2
Beginning Balance			1,061.63	552.63
Interest Accrued			91.00	47.37
Payment to debt holders			600.00	600.00
Ending Balance		1,061.63	552.63	0.00

The interest payment for year n is based on the loan balance at the beginning of year n.

Based on the present values of the cash flows to the levered equity holders, we can calculate the debt-equity ratios at the end of year 0 and year 1. And using the values of the percent debt and percent equity in table 6, the return to levered equity and the WACC for year 1 and year 2 can be calculated. The annual returns to unlevered equity in year 1 and year 2 are equal to the (pre-tax) WACCs in year 1 and year 2, respectively. See line C3.3 and line C4.3 in Appendix C for the calculation of the WACCs. The return to unlevered equity in year 1, ρ_1, is 16.18% and the return to unlevered equity in year 2, ρ_2, is 17.94%.

	Year	0	1	2
Percent Debt			33.41%	18.03%
Percent Equity			66.59%	81.97%
Debt-Equity ratio			0.502	0.220

Table 6: Debt-equity ratios at the end of year 0 and year 1

Based on the returns to unlevered equity in year 1 and year 2, we can calculate the present values of the tax savings in at the end of year 0 and year 1. See line C6.1 and line C6.2 in Appendix C.

	Year	0	1	2
Tax Savings			27.30	14.21
Present Value of Tax Savings		33.86	12.05	

Table 7: The tax savings and the present value of the tax shield

The tax savings in year 1 and year 2 are $27.30 and $14.21, respectively; the present value of the tax shield at the end of year 0 is $33.56 and the present value of the tax shield at the end of year 1 is $12.10. See line C1.1 and line C1.2 in Appendix C.

With the tax savings, the cash flows to the debt and equity holders are given in the following table. In any year n, the cash flow to the equity holder (levered) includes the tax savings in year n.

	Year	0	1	2
Free Cash Flow (FCF)			600.0	3,600.0
Tax Savings			27.3	14.2

	Year	0	1	2
Total Cash Flow			627.3	3,614.2
Cash Flow to debt holders (CFD)			600.0	600.0
Cash Flow to equity holders (CFE)			27.3	3,014.2

Table 8: Cash Flows to the debt and equity holders

The values of debt and levered equity are summarized in the following table. In any year n, the value of the levered cash flows is equal to the value of the unlevered cash flows plus the present value of the tax shield. In addition, in any year n, the value of the levered cash flows is equal to the value of the cash flows to the debt holders and the value of the cash flows to the equity holders.

	Year	0	1	2
Value of debt		1,061.63	552.63	
Value of levered equity		2,115.95	2,511.84	
Total levered Value		3,177.59	3,064.47	
PV of Tax Shield		33.87	12.05	
Value of unlevered equity		3,143.72	3,052.42	

Table 9: Value of equity (levered) and debt

The WACC for year 1 and year 2 are 16.182% and 17.939% respectively. See line C3.3 and line C4.3 in Appendix C. The return to unlevered equity in year 1 and year 2 are equal to the WACC in year 1 and year 2, respectively, and are shown in the following table.

	Year	0	1	2
Cost of debt			8.6%	8.6%
Return to levered equity			20.0%	20.0%
WACC (pre-tax)			16.182%	17.939%
Return to unlevered equity			16.182%	17.939%

Table 10: Return to levered equity and WACC for year 1 and year 2

Discrepancies in the value estimates from the Discounted Cash Flow Model

Now we are in a position to estimate the magnitude of the errors in the value estimates obtained from the DCF model relative to the RI model. The size of the discrepancy will depend on the assumptions that are used in the estimation of the WACC.

As a point of reference, we will assume that at the end of year 0, the **correct** present value of the cash flows to the levered equity holder is $2,116 and the present value of the levered cash flow (namely the debt plus equity) is $3,177.6. See Table 9 and Summary Table C in Appendix C.

Alternative ways to calculate the after-tax WACC

The after-tax WACC will only be constant if the debt (as a percent of the levered value) is constant for the two periods, but in this numerical example, the percent of debt changes from year 1 to year 2. In fact, we know that the correct value for the after-tax WACC in year 1 is 15.32% and the correct after-tax WACC in year 2 is 17.48%. See footnote #5. For the calculation of the after-tax WACCs, see line C3.4 and line C4.4 in Appendix C.

We will examine two alternative ways for calculating the values of the WACC. First, we could use the average percentage of debt over the two periods to calculate the WACC, in which case the appropriate WACC is 16.40%. See line C14.3 in Appendix C. Second, we could find the single WACC that would be equivalent to the multiple WACCs, in which case the appropriate WACC is 16.30%. See line C15.2 in Appendix C.

The WACC of 16.40%, based on the average debt and equity percentages for the two years, is very close to the single WACC of 16.30%.

WACC	Total Levered Value		Levered Equity	Diff = Lev. Equity - 2,116	%Diff
15.00%	3,243.86		2,182.22	66.27	3.13%
15.20%	3,233.51		2,171.87	55.92	2.64%
15.40%	3,223.21		2,161.58	45.62	2.16%

WACC	Total Levered Value		Levered Equity	Diff = Lev. Equity - 2,116	%Diff
15.60%	3,212.96		2,151.33	35.38	1.67%
15.80%	3,202.77		2,141.14	25.19	1.19%
16.00%	3,192.63		2,130.99	15.04	0.71%
16.20%	3,182.54		2,120.90	4.95	0.23%
16.40%	3,172.49		2,110.86	-5.09	-0.24%
16.60%	3,162.50		2,100.87	-15.08	-0.71%
16.80%	3,152.56		2,090.93	-25.02	-1.18%
17.00%	3,142.67		2,081.03	-34.92	-1.65%

Table 11: Relationship between values for the WACC and the total levered values and levered equity values.

Table 11 shows relationship between different values for the WACC and the total levered values and the levered equity values. Based on Table 11, we can determine the absolute difference and percentage difference between the value estimates from the DCF model and the RI model relative the values obtained from the RI model. In addition to these two values for the WACC, we could have used the debt percentage in the first year or the second year. For example, if we used a single WACC based on the debt percentage in the first year, the value of the WACC would be 15.32%, the value of the levered equity value would be overstated by \$50, and the percentage difference would approximately 2.5%.

Conclusion

In principle, the value estimates from the Discounted Cash Flow Model and the Residual Income Model must give the same results if the valuation is consistently conducted in an M & M world. In practice, the use of a constant return to levered equity (with the implicit assumption that ρ, the return to unlevered equity, is constant) in the presence of changing debt-equity ratios will introduce errors in the value estimates obtained from the Residual Income Model. In the Discounted Cash Flow Model, the sources of the errors lie in the assumptions about the annual percentage of debt and the annual return to levered equity (as a function of the debt-equity ratio). To be consistent with an M & M world, the annual WACC must take into account the changing percent of debt and the changing return to levered equity.

References

LUNDHOLM, R. and O'KEEFE, T. 2000. "Reconciling Value Estimates from the Discounted Cash Flow Value Model and the Residual Income Model". Working Paper, Social Science Research Network (SSRN).

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APPENDIX A: Without taxes

Assumption: The return to unlevered equity is constant.

1. The return to levered equity in year 2 is 20%, and the corresponding return to unlevered equity in year 2 is 17.42%.
2. In year 1, the return to unlevered equity in year 1 is 17.42%. The return to levered equity will change to 23.63%

Value of debt at the end of year 0 and year 1, with no tax

After-tax Cost of debt, d*(1 - t) = 6%, where d = 6% and t = 0%. The cost of debt is assumed to be constant for year 1 and year 2. CFD_1 is the cash flow to the debt holders in year 1 and CFD_2 is the cash flow to the debt holders in year 2.

The present value of the debt at the end of year 0 is

$$D_0 = \frac{CFD_1}{1+d} + \frac{CFD_2}{(1+d)^2} = \frac{600}{1+6\%} + \frac{600}{(1+6\%)^2} = 566.04 + 534.00 = 1{,}100.04 \qquad \text{(A1.1)}$$

$$D_1 = \frac{600}{1+6\%} = 566.04 \qquad \text{(A1.2)}$$

Value of levered Equity

We will assume that e_2, the return to levered equity in year 2 is 20%, based on the debt-equity ratio at the beginning of year 2. CFE_2 is the cash flow to the equity holders in year 2.

$$E_1 = \text{PV of levered equity (end of yr 1)} = \frac{CFE_2}{1+e_2} = \frac{3{,}000}{1+20\%} = 2{,}500.00 \qquad \text{(A2)}$$

Determination of the return to unlevered equity

Let ρ = return to unlevered equity. It is assumed to be constant for year 1 and year 2.

Assumption: M & M's Theorem (with no taxes) holds at the end of year 0 and end of year 1.

$$V^{UL}_1 = V^L_1 = E_1 + D_1 \quad \text{(A3.1)}$$

$$V^{UL}_0 = V^L_0 = E_0 + D_0 \quad \text{(A3.2)}$$

where V^{UL}_n is the present value of the unlevered cash flow in year n and V^L_n is the present value of the levered cash flow in year n. And

$$e_n = \rho + (\rho - d)*\frac{D_{n-1}}{E_{n-1}} \quad \text{(A4)}$$

where e_n is the return to levered equity in year n. The values for debt and equity are based on the values at the beginning of year n (or equivalently at the end of year n-1, using the end of year convention throughout)

Solving for ρ, we obtain that

$$\rho = \frac{e*E_n + d*D_n}{E_n + D_n} \quad \text{(A5)}$$

Return to levered equity in year 2

$$e_2 = \rho + (\rho - d)*\frac{D_1}{E_1} = 20\% \quad \text{(A6)}$$

We have assumed that e_2 is 20%.

Return to unlevered equity in year 1

$$\rho = \frac{e*E_1 + d*D_1}{E_1 + D_1}$$

$$= \frac{20\%*2{,}500 + 6\%*566}{2{,}500 + 566} = 17.42\% \quad \text{(A7)}$$

Value of unlevered equity, end of year 0

FCF_1 is the free cash flow at the end of year 1 and FCF_2 is the free cash flow at the end of year 2

$$V^{UL}_0 = \text{PV of FCF (end of yr 0)} = \frac{FCF_1}{1+\rho} + \frac{FCF_2}{(1+\rho)^2}$$

$$= \frac{600}{1 + 17.42\%} + \frac{3{,}600}{(1 + 17.42\%)^2}$$

$$= 510.99 + 2{,}611.07 = 3{,}122.3 \quad \text{(A8.1)}$$

$$V^{UL}_1 = \text{PV of FCF (end of yr 1)} = \frac{FCF_2}{1+\rho}$$

$$= \frac{3{,}600.0}{1 + 17.42\%} = 3{,}066.0 \quad \text{(A8.2)}$$

Value of levered equity at the end of year 0?

$$E_0 = V^{UL}_0 - D_0 = 3{,}122 - 1{,}100 = 2{,}022.00 \quad \text{(A9)}$$

Return to levered equity in year 1

$$e_1 = \rho + (\rho - d)*\frac{D_0}{E_0}$$

$$= 17.42\% + (17.42\% - 6\%)*\frac{1{,}100}{2{,}022} = 23.63\% \quad \text{(A10)}$$

CFE_1 is the cash flow to equity holders at the end of year 1 and CFE_2 is the cash flow to equity holders at the end of year 2

$$E_0 = \text{PV of CFE (end of yr 0)} = \frac{CFE_1}{1+e_1} + \frac{CFE_2}{(1+e_1)*(1+e_2)}$$

$$= \frac{0.00}{1 + 23.63\%} + \frac{3{,}000}{(1 + 23.63\%)*(1 + 20\%)}$$

$$= 2{,}022.16 \quad \text{(A11)}$$

Weighted Average Cost of Capital in Year 2

$WACC_2$ = Weighted average cost of capital in year 2

$$\%D_1 = \frac{D_1}{E_1 + D_1} = \frac{566}{2{,}500 + 566} = 18.46\% \quad \text{(A12.1)}$$

$$\%E_1 = \frac{E_1}{E_1 + D_1} = \frac{2{,}500}{2{,}500 + 566} = 81.54\% \quad \text{(A12.2)}$$

$$WACC_2 = \%D_1*d + \%E_1*e_2$$

$$= 18.46\%*6\% + 81.54\%*20\% = 17.42\% \quad \text{(A12.3)}$$

Weighted Average Cost of Capital in Year 1

$WACC_1$ = Weighted average cost of capital in year 1

$$\%D_0 = \frac{D_0}{E_0 + D_0} = \frac{1{,}100}{2{,}022 + 1{,}100} = 35.23\% \quad \text{(A13.1)}$$

$$\%E_0 = \frac{E_0}{E0 + D0} = \frac{2{,}022}{2{,}022 + 1{,}100} = 64.77\% \quad \text{(A13.2)}$$

$$WACC_1 = \%D_0{*}d + \%E_0{*}e_1$$
$$= 35.23\%{*}6\% + 64.77\%{*}23.63\% = 17.42\% \quad \text{(A13.3)}$$

Note that the return to unlevered equity is constant and equal to the WACC in both years.

Summary Table A: Constant Return to unlevered equity

	Year	0	1	2
Free Cash Flow (FCF)			600.0	3,600.0
Cash Flow to debt holders (CFD)			600.0	600.0
Cash Flow to equity holders (CFE)			0.0	3,000.0
	Year	0	1	2
Value of debt		1,100.0	566.0	
Value of equity (levered)		2,022.2	2,500.0	
Total Value = debt + equity (levered)		3,122.3	3,066.0	
	Average	0	1	2
Percent Debt	26.85%		35.23%	18.46%
Percent Equity	73.15%		64.77%	81.54%
Debt-Equity Ratio	0.367		0.544	0.226
	Year	0	1	2
Cost of debt			6.0%	6.0%
Return to levered equity			23.6%	20.0%
WACC			17.415%	17.415%
Return to unlevered equity			17.415%	17.415%

Value of WACC based on the average percent debt over the two years

$$\text{Average Percent Debt} = \frac{35.23\% + 18.46\%}{2} = 26.85\% \quad \text{(C14.1)}$$

$$\text{Average Percent Equity} = \frac{64.77\% + 81.54\%}{2} = 73.16\% \quad \text{(C14.2)}$$

$$WACC = \%D{*}d + \%E{*}e_2$$
$$= 26.85\%{*}6\% + 73.16\%{*}20\% = 16.24\% \quad \text{(C14.3)}$$

Value of WACC based on the percent of debt in year 1

$$WACC = \%D_1{*}d + \%E_1{*}e_2$$
$$= 35.23\%{*}6\% + 64.77\%{*}20\% = 15.07\% \quad \text{(C15)}$$

Value of WACC based on the percent of debt in year 2

$$WACC = \%D_1{*}d + \%E_1{*}e_2$$
$$= 18.46\%{*}6\% + 81.54\%{*}20\% = 17.42\% \quad \text{(C16)}$$

APPENDIX B: without taxes

Assumption: The return to levered equity is constant.

1. The return to levered equity in year 2 is 20%, and the corresponding return to unlevered equity in year 2 is 17.415%.
2. In year 1, the return to levered equity is 20%. The return to unlevered equity will change to

$$E_0 = \text{PV of CFE (end of yr 0)} = \frac{CFE_1}{1+e} + \frac{CFE_2}{(1+e)^2}$$
$$= \frac{0.00}{1+20\%} + \frac{3{,}000}{(1+20\%)^2}$$
$$= 2{,}083.33 \quad \text{(B1)}$$

Value of unlevered equity at the end of year 0?

$$V^{UL}_0 = E_0 + D_0 = 2{,}083.33 + 1{,}100 = 3{,}183.33 \quad \text{(B2)}$$

Weighted Average Cost of Capital in Year 1 with the values in table 2a

$WACC_1$ = Weighted average cost of capital in year 1

$$\%D_0 = \frac{D_0}{E_0 + D_0} = \frac{1{,}100}{2{,}083 + 1{,}100} = 34.56\% \quad \text{(B3.1)}$$

$$\%E_0 = \frac{E_0}{E_0 + D_0} = \frac{2{,}083}{2{,}083 + 1{,}100} = 65.44\% \quad \text{(B3.2)}$$

$$WACC_1 = \%D_0{*}d + \%E_0{*}e_1$$
$$= 34.56\%{*}6\% + 65.44\%{*}20\% = 15.16\% \quad \text{(B3.3)}$$

If we assume that the return to levered equity is constant at 20% for year 1 and year 2, we can find the WACC for the free cash flow. The WACC is the IRR for the cash flow.

$$V^L_0 = 3{,}183.4 = \frac{FCF_1}{1 + WACC} + \frac{FCF_2}{(1 + WACC)^2} \qquad (B4.1)$$

We can verify that

$$\frac{600}{1 + 16.18\%} + \frac{3{,}600}{(1 + 16.18)^2} \qquad (B4.2)$$

$$= 516.43 + 2{,}666.97 = 3{,}183.4$$

Summary Table B: Constant return to levered equity

	Year	0	1	2
Free Cash Flow (FCF)			600.0	3,600.0
Cash Flow to debt holders (CFD)			600.0	600.0
Cash Flow to equity holders (CFE)			0.0	3,000.0
	Year	0	1	2
Value of debt		1,100.0	566.0	
Value of equity (levered)		2,083.3	2,500.0	
Total Value = debt + equity (levered)		3,183.4	3,066.0	
	Average	0	1	2
Percent Debt	26.51%		34.56%	18.46%
Percent Equity	73.49%		65.44%	81.54%
Debt-Equity Ratio	0.361		0.528	0.226
	Year	0	1	2
Cost of debt			6.0%	6.0%
Return to levered equity			20.0%	20.0%
WACC			15.162%	17.415%

	Year	0	1	2
Return to unlevered equity			15.162%	17.415%

APPENDIX C: with taxes

Assumption: The return to levered equity is constant.

1. The return to levered equity in year 2 is 20%, and the corresponding return to unlevered equity in year 2 is 17.94%.

Value of debt at the end of year 0 and year 1, with tax

After-tax Cost of debt, d*(1 - t) = 6%, where d = 8.57% and t = 30%. The cost of debt is assumed to be constant for year 1 and year 2. The present value of the debt at the end of year 0 is

$$D_0 = \frac{CFD_1}{1 + d} + \frac{CFD_2}{(1 + d)^2}$$

$$= \frac{600}{1 + 8.57\%} + \frac{600}{(1 + 8.57\%)^2}$$

$$= 552.64 + 509.02 = 1{,}061.66 \qquad (C1.1)$$

$$D_1 = \frac{600}{1 + 8.57\%}$$

$$= 552.64 \qquad (C1.2)$$

Value of levered equity at the end of year 1 and year 0

The present value (end of year 1) of the cash flows to the equity holder

$$E_1 = \frac{FCF_2 - CFD_2 + TS_2}{1 + 20\%}$$

$$= \frac{3{,}600 - 600 + 14.21}{1 + 20\%}$$

$$= 2{,}511.84 \qquad (C2.1)$$

The present value (end of year 0) of the cash flows to the equity holder

$$E_0 = \frac{FCF_1 - CFD_1 + TS_1}{1 + e} + \frac{FCF_2 - CFD_2 + TS_2}{(1 + e)^2}$$

$$= \frac{600 - 600 + 27.2}{1 + 20\%} + \frac{3{,}600 - 600 + 14.21}{(1 + 20\%)^2}$$

$$= 22.75 + 2{,}093.19 = 2{,}115.9 \qquad \text{(C2.2)}$$

Weighted Average Cost of Capital in Year 2 (pre-tax)

$WACC_2$ = Weighted average cost of capital in year 2

$$\%D_1 = \frac{D_1}{E_1 + D_1} = \frac{552.6}{2{,}511.84 + 552.6} = 18.03\% \qquad \text{(C3.1)}$$

$$\%E_1 = \frac{E_1}{E_1 + D_1} = \frac{2{,}525.5}{2{,}511.84 + 552.6} = 81.97\% \qquad \text{(C3.2)}$$

$$WACC_2 = \%D_1{*}d + \%E_1{*}e_2$$
$$= 18.03\%{*}8.57\% + 81.97\%{*}20\% = 17.94\% \qquad \text{(C3.3)}$$

The return to unlevered equity ρ_2 is equal to the $WACC_2$ (pre-tax)

$$\text{(After-tax) } WACC_2 = \%D_1{*}d{*}(1 - t) + \%E_1{*}e_2$$
$$= 18.03\%{*}6\% + 81.97\%{*}20\% = 17.48\% \qquad \text{(C3.4)}$$

Weighted Average Cost of Capital in Year 1 (pre-tax)

$WACC_1$ = Weighted average cost of capital in year 1

$$\%D_0 = \frac{D_0}{E_0 + D_0} = \frac{1{,}061.6}{2{,}115.95 + 1{,}061.6} = 33.41\% \qquad \text{(C4.1)}$$

$$\%E_0 = \frac{E_0}{E_0 + D_0} = \frac{2{,}115.95}{2{,}115.95 + 1{,}061.6} = 66.59\% \qquad \text{(C4.2)}$$

$$WACC_1 = \%D_0{*}d + \%E_0{*}e_1$$
$$= 33.41\%{*}8.57\% + 66.59\%{*}20\% = 16.18\% \qquad \text{(C4.3)}$$

The return to unlevered equity r_1 is equal to the $WACC_1$ (pre-tax)

$$\text{(After-tax) } WACC_1 = \%D_0{*}d + \%E_0{*}e_1$$
$$= 33.41\%{*}6\% + 66.59\%{*}20\% = 15.32\% \qquad \text{(C4.4)}$$

Tax Savings in year 1 and year 2

TS_n, the tax savings in year n is equal to the product of the interest payments in year n and the tax rate. TS_1 is the tax savings in year 1 and TS_2 is the tax savings in year 2.

$$TS_2 = t{*}d{*}D_1 = 30\%{*}8.57\%{*}552.6 = 14.21 \qquad \text{(C5.1)}$$

$$TS_1 = t{*}d{*}D_0 = 30\%{*}8.57\%{*}1{,}061.6 = 27.29 \qquad \text{(C5.2)}$$

Present value of tax shield at the end of year 0 and year 1

The discount rate for the tax shield is ρ, the return to unlevered equity and is equal to 17.42%.

The present value of the tax shield at the end of year 0 is

$$V^{TS}_0 = \text{PV of TS (end of yr 0)} = \frac{TS_1}{(1 + \rho_1)} + \frac{TS_2}{(1 + \rho_1){*}(1 + \rho_2)}$$
$$= \frac{27.30}{1 + 16.182\%} + \frac{14.21}{(1 + 16.182\%){*}(1 + 17.939\%)}$$
$$= 23.50 + 10.36 = 33.86 \qquad \text{(C6.1)}$$

The present value of the tax shield at the end of year 1 is

$$V^{TS}_1 = \text{PV of TS (end of yr 1)} = \frac{TS_2}{(1 + \rho_1)}$$
$$= \frac{14.21}{1 + 17.939\%}$$
$$= 12.05 \qquad \text{(C6.2)}$$

Present value of the unlevered cash flow at the end of year 0 and year 1

In year 1, the return to unlevered equity is 16.182% and in year 2, the return to unlevered equity is 17.939%.

Present value of the unlevered cash flow at the end of year 0

$$V^{UL}_0 = \text{PV of FCF (end of year 0)} = \frac{FCF_1}{(1 + \rho_1)} + \frac{FCF_2}{(1 + \rho_1){*}(1 + \rho_2)}$$
$$= \frac{600}{1 + 16.182\%} + \frac{3{,}600}{(1 + 16.182\%){*}(1 + 17.939\%)}$$
$$= 516.43 + 2{,}627.28 = 3{,}143.71 \qquad \text{(C7.1)}$$

$$V^{UL}_1 = \text{PV of FCF (end of yr 1)} = \frac{FCF_2}{(1 + \rho_1)}$$
$$= \frac{3{,}600}{1 + 17.939\%}$$
$$= 3{,}052.43 \qquad \text{(C7.2)}$$

Reconciliation in an M & M world (1)

For any year n, the FCF discounted at the $WACC_n$ (after-tax) is equal to the FCF discounted at ρ plus the tax savings discounted at ρ.

$$V^L_0 = V^{UL}_0 + V^{TS}_0 \quad (C8.1)$$

$$V^L_1 = V^{UL}_1 + V^{TS}_1 \quad (C8.2)$$

$$V^L_0 = \text{PV of FCF (end of yr 0)} = \frac{FCF_1}{1 + WACC_1} + \frac{FCF_2}{(1 + WACC_1)*(1 + WACC_2)}$$
$$= \frac{600}{1 + 15.323\%} + \frac{3,600}{(1 + 15.323\%)*(1 + 17.475\%)}$$
$$= 520.28 + 2,657.30 = 3,177.58 \quad (C9.1)$$

$$V^L_1 = \text{PV of FCF (end of yr 0)} = \frac{FCF_2}{1 + WACC_2}$$
$$= \frac{3,600}{1 + 17.475\%} = 3,064.48 \quad (C9.2)$$

$$V^L_0 = 3,177.58 = V^{UL}_0 + V^{TS}_0 = 3,143.72 + 33.87 = 3,177.59 \quad (C10.1)$$

$$V^L_1 = 3,064.48 = V^{UL}_1 + V^{TS}_1 = 3,052.42 + 12.05 = 3,064.5 \quad (C10.2)$$

Summary Table C: Constant return to levered equity, with taxes

	Year	0	1	2
Free Cash Flow (FCF)			600.0	3,600.0
Cash Flow to debt holders (CFD)			600.0	600.0
Tax savings			27.30	14.21
Cash Flow to equity holders (CFE)			0.0	3,000.0
	Year	0	1	2
Value of debt		1,061.6	552.6	
Value of equity (levered)		2,116.0	2,511.8	
Total Value = debt + equity (levered)		3,177.6	3,064.5	
	Year	0	1	2
PV of Tax Shield		33.87	12.05	
Value of unlevered equity		3,143.72	3,052.42	

	Average	0	1	2
Percent Debt	25.72%		33.41%	18.03%
Percent Equity	74.28%		66.59%	81.97%
Debt-Equity Ratio	0.346		0.502	0.220
	Year	0	1	2
Cost of debt			8.6%	8.6%
Return to levered equity			20.0%	20.0%
WACC (pre-tax)			16.182%	17.939%
Return to unlevered equity			16.182%	17.939%
WACC (after-tax)			15.323%	17.475%

Reconciliation in an M & M world (2)

For any year n, the FCF discounted at the $WACC_n$ (after tax) is equal to the CFE discounted at e_n plus the CFD discounted at d. That is,

$$V^L_0 = E_0 + D_0 \quad (C11.1)$$

$$V^L_1 = E_1 + D_1 \quad (C11.2)$$

$$E_0 = \text{PV of CFE (end of yr 0)} = \frac{CFE_1}{1 + e} + \frac{CFE_2}{(1 + e)^2}$$
$$= \frac{27.3}{1 + 20\%} + \frac{3,014.2}{(1 + 20\%)^2}$$
$$= 22.75 + 2,093.19 = 2,115.94 \quad (C12.1)$$

$$E_1 = \text{PV of CFE (end of yr 1)} = \frac{CFE_2}{1 + e}$$
$$= \frac{3,014.2 = 2,511.83}{1 + 20\%} \quad (C12.2)$$

$$V^L_0 = 3,177.59 = E_0 + D_0 = 2,115.94 + 1,061.6 = 3,177.5 \quad (C13.1)$$

$$V^L_1 = 3,064.47 = E_1 + D_1 = 2,511.83 + 552.6 = 3,064.4 \quad (C13.2)$$

Value of WACC based on the average percent debt over the two years

$$\text{Average Percent Debt} = \frac{33.42\% + 18.03\%}{2} = 25.73\% \quad \text{(C14.1)}$$

$$\text{Average Percent Equity} = \frac{66.59\% + 81.97\%}{2} = 74.28\% \quad \text{(C14.2)}$$

$$\text{WACC} = \%D*d*(1 - t) + \%E*e_2$$
$$= 25.73\%*6\% + 74.28\%*20\% = 16.40\% \quad \text{(C14.3)}$$

Single WACC equivalent to the two multiple WACCs over the two years

$$V^L_0 = 3{,}177.6 = \frac{FCF_1}{1 + WACC} + \frac{FCF_2}{(1 + WACC)^2} \quad \text{(C15.1)}$$

We can verify that

$$\frac{600}{1 + 16.30\%} + \frac{3{,}600}{(1 + 16.30\%)^2} \quad \text{(C15.2)}$$
$$= 515.91 + 2{,}661.60 = 3{,}177.5$$