Common errors regarding terminal value perpetuities[1]

Errores comunes relativos al valor terminal expresado en forma de perpetuidad

Erros comuns relativos ao valor terminal expressado em forma de perpetuidade

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Abstract

In firm valuation the free-cash flows after the horizon are usually represented by a terminal value. In this paper attention is centered on the terminal value expressed as the present value of perpetuity. Seven important mistakes have been identified when working with these perpetuities. They are mainly caused by the following misjudgments: a) Failure to correctly account for the instability of the free-cash flows during the projection horizon; b) Lack of consistency between the last free-cash flow of the horizon and the assumptions about reinvestment and growth; c) Failure to account for the possibility to grow through external funding; d) Inconsistency between the market exchange rate and the equilibrium exchange rate when working in different currencies; e) Ignoring the possibility of different phases of growth; and f) Setting up growth rates with little concern for the characteristics of the firm and the industry, inflation, market growth or market share.

Key Words: Terminal Values, Perpetuities, Valuation, Free-Cash Flows

Resumen

Al valorar una firma los flujos de caja libres después del horizonte de proyección son generalmente representados por un valor terminal. Este trabajo centra su atención en el valor terminal expresado como el valor presente de una perpetuidad. Siete importantes errores han sido identificados cuando se trabaja con dichas perpetuidades. Las principales causas de dichos errores son: a) Desestimar la inestabilidad de los flujos de caja libres durante el horizonte de proyección; b) Inconsistencia entre el último flujo de caja libre del horizonte de proyección y los supuestos sobre reinversión y crecimiento; c) Ignorar la posibilidad de crecimiento a través de financiamiento exógeno; d) Inconsistencia entre la tasa de cambio de mercado y la tasa de equilibrio cuando se trabaja con diferentes monedas; e) Ignorar la posibilidad de varias fases de crecimiento; y f) Establecer la tasa de crecimiento sin la debida consideración a las características de la firma y de la industria, la inflación, y el crecimiento y cuota de mercado.

Palabras clave: valores terminales, perpetuidades, valoración, flujo de caja libre.

Resumo

Uma firma, ao avaliar os fluxos de caixa livres depois do horizonte de projeção, encontra que estes são geralmente representados por um valor terminal. Este trabalho centra sua atenção no valor terminal que é expressado como o valor presente de uma perpetuidade. Sete importantes erros foram identificados quando se trabalha com estas perpetuidades. As principais causas de ditos erros são: a) Desestimar a instabilidade dos fluxos de caixa livres durante o horizonte de projeção; b) Inconsistência entre o último fluxo de caixa livre do horizonte de projeção e os pressupostos sobre reinvestimento e crescimento; c) Ignorar a possibilidade de crescimento através do financiamento exógeno; d) Inconsistência entre a taxa de câmbio de mercado e a taxa de equilíbrio quando se trabalha com diferentes moedas; e) Ignorar a possibilidade de várias fases de crescimento; e f) Estabelecer a taxa de crescimento sem a devida consideração das características da firma e da indústria, a inflação e o crescimento e quota de mercado.

Palavras-chave: valores terminais, perpetuidades, valoração, fluxo de caixa livre.

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Introduction

On most discounted cash flow valuations of companies a terminal value is assumed at the end of the projection horizon. There are several methods for the computation of the terminal value. The most common are: a) assuming that the company will cease to exist and estimating a liquidation value; b) calculating a multiple based on comparable firms and referred to key variables such as Sales, Earnings Before Interest and Taxes (EBITDA), Volume of Production, etc. and; c) assuming that the company will operate "forever" and thus calculating the final value as the present value of a perpetuity.

In this paper attention is centered on the terminal value expressed as the present value of perpetuity. Different aspects pertaining this perpetuity are discussed and a number of errors commonly made by analysts are pinpointed[3].

The topics to be covered are:

- The relationship between the terminal value and the period of time during which the cash flows are explicitly projected.
- The calculation of the first free-cash flow of the perpetuity defining the terminal value.
- Growth through external financing.
- The impact of exchange rates on terminal value.
- The determinants of long-term growth.

The purpose is to discuss how these different issues might affect the final result of a valuation. Of course, it must be remembered that valuations are not but an exercise to estimate the possible value of a company, and that a well-performed valuation will never produce the "true" value of a firm but just a range of values consistent with the assumptions adopted by the analyst. This paper only pretends to improve the estimation of such a range. For simplicity, through most of the paper parameters and projections will ignore inflation. That is, they will be expressed in real terms.

Horizon and Terminal Value

In this document the horizon is understood as the period of time during which cash flows are projected in detail. After this period presumably some type of stability is achieved and a perpetuity is specified. Naturally, the longer the horizon, the lower the relative weight of the terminal value in the total value of the firm, and vice versa.

It is considered that stability takes place when all the parameters defining free-cash flows, such as margins and balance sheet ratios, are supposed constant from that point on. Every time a perpetuity is assumed, even if it is not constant but growing, decreasing or of any other type, there is a presumption of stability as stated in this way (even though free-cash flows might vary over time).

Of course, no company operates for infinity. The perpetuity is simply a proxy for a "long" period of time and, given the decreasing importance in present value terms of those free-cash flows taking place very far into the future, the proxy ends up being a good approximation of reality.

Given that the terminal value requires assumptions (i.e. the ratios defining free-cash flows) that will hold for a long period of time (theoretically up to infinity), it is only logical to project cash flows for as long as possible so that the weight of the terminal value and the relevance of its assumptions are less critical to the final result.

The practical difficulty of this reasoning is that the assumptions behind the explicit cash flows become less and less reliable as the horizon is extended. This is why, when faced with uncertain environments many analysts prefer to shorten the horizon. However, this poses a problem. A shortened horizon demands defining a perpetuity earlier, thus accepting a set of fixed assumptions forever. Paradoxically, when faced with uncertainty, a shorter period of instability (the horizon) and a longer period of stability (the perpetuity) are chosen. Thus, the preferred "solution" is to increase certainty![4]

Obviously, shortening the horizon is not a consistent approach. Instead, it is better to deal with uncertain cash flows for as long as possible and ponder the effects of uncertainty through other means such as setting several future scenarios of different likelihood, performing sensitivity analyses to the more sensible variables, or carrying out Monte Carlo simulations. The final result will be a range of reasonable values for the firm[5].

3. I thank Carmen Ansotegui, Maximiliano Gonzalez, Urbi Garay and anonymous reviewers for very helpful comments.

4. It must be clarified that uncertainty is understood here as different from risk. A risky variable can be statistically modeled whereas an uncertain one cannot because not enough information is available for assigning a particular distribution function. When dealing with uncertain variables any scenarios or simulation will thus demand a degree of subjectivity from the analyst.

5. The discount rate as estimated through the popular CAPM method takes risk into account but not uncertainty. Because of this, uncertainty cannot be accounted for in the discount rate but only in the cash flows.

As a general rule, the projection must be for as long a period as possible and a perpetuity established only at the point when a high degree of stability (in the sense stability was defined before) can be presumed for all future cash flows taking place after the horizon.

Moreover, given that the perpetuity´s assumptions apply for a very long time it is safer to make sure that its final weight on the total value of the project is not too significant. A common "rule of thumb" is for the perpetuity not to represent more than 30% of the total valuation.

ERROR 1: Instead of dealing directly with the effects of uncertainty, paradoxically the analyst ignores uncertainties and assumes more stability by shortening the horizon and defining a perpetuity too early in time.

The First Cash Flow of the Perpetuity

In this section the computation of the first free-cash flow of the perpetuity is discussed. The analysis is based on unlevered free-cash flows after taxes.

The general expression for the computation of a free-cash flow is:

$$FCF\ EBIT + DA - T - CAPEX - INWC \quad (1)$$

where,

EBIT = Earnings before interest and taxes.

DA = Depreciation and amortization.

T = Taxes.

CAPEX = Capital expenditures.

INWC = Investment in net working capital.

If the horizon lasts n years, for the first year of the perpetuity the free-cash flow is[6],

$$FCF_{n+1} = EBIT_{n+1} + DA_{n+1} - T_{n+1} - CAPEX_{n+1} - INWC_{n+1} \quad (2)$$

Let us focus first on the case where there is no growth. For no growth, capital expenditures must equal depreciation and amortization allowances in every year. Also, no additional investments in working capital are needed. Thus,

$$CAPEX_{n+1} = DA_{n+1} \text{ and } INWC_{n+1} = 0 \quad (3)$$

Therefore, the first free-cash flow of the perpetuity is simplified as,

$$FCF_{n+1} = EBIT_{n+1} - T_{n+1} \quad (4)$$

6. Yearly cash flows are supposed, but clearly the same rationale applies for different periods such as semesters or months.

This expression is also known as "Net Operating Profit after Taxes" (*NOPAT*). Thus,

$$NOPAT_{n+1} = EBIT_{n+1} - T_{n+1} \quad (5)$$

NOPAT equals the earnings after taxes are paid out. So, it can be interpreted as the cash flow received by all fund providers, meaning shareholders and bondholders.

In sum, $NOPAT_{n+1}$ is the free-cash flow of the first year of the perpetuity when there is no growth (Koller, Goedhart & Wessels 2010).

Now, let us focus on the more common case where there is some growth after the horizon. As opposed to the no-growth case discussed before, for positive growth it is necessary to undertake capital expenditures beyond depreciation and amortization and invest in net working capital as well. As a result,

$$FCF_{n+1} = NOPAT_{n+1} + DA_{n+1} - CAPEX_{n+1} - INWC_{n+1} \quad (6)$$

where investment in net working capital ($INWC_{n+1}$) depends on operational growth, and

$$CAPEX_{n+1} > DA_{n+1}\ .$$

Beware that *CAPEX* must be defined as expenses designed to generate benefits over many years. So, they should include research and development, training and recruiting costs and exploration expenditures (in the case of mining companies). All these expenses must be capitalized and depreciated (Damodaran 1999).

The "reinvestment ratio" (*b*) corresponds to the proportion of the free-cash flow that must be reinvested year after year to assure growth. It is well-known that the reinvestment ratio (*b*) can be expressed as:

$$b = \frac{g}{r} \quad (7)$$

where

g = the firm´s growth rate and,

r = return on reinvested assets

The "return on reinvested assets" (*r*) is the rate of return that is expected to be sustained on the reinvested free-cash flows in perpetuity, after the horizon.

By definition all the relationships in any perpetuity must remain constant. In consequence, the "reinvestment ratio" (*b*) must comply with[7],

7. The proportion or earnings that are reinvested is commonly known as the "plow-back" ratio. The ratio (*CAPEX -DA+ INWC)/ NOPAT* is similar to the plow-back ratio, but in this case applies to unlevered free-cash flows.

$$b = \frac{CAPEX_t - DA_t + INWC_t}{NOPAT_t} = \text{constant for } t \geq n. \quad (8)$$

The amounts reinvested will normally be financed via a combination of debt and equity according to the capital structure selected for the firm.

ERROR 2: It is common to find valuations where the first cash flow of the perpetuity is erroneously computed as the last cash flow of the horizon multiplied by 1 plus a growth rate. The error consists in that the last cash flow of the horizon does not account for any permanent reinvestment and/or the growth rate is independent from the reinvestment ratio *b* and the return on reinvested assets *r*. The final result is a considerable overestimation of firm value.

The practice mentioned in ERROR 2 is right only as long as FCF_n is consistent with long-term reinvestment and growth. The correct expression for the first free-cash flow after the horizon is:

$$FCF_{n+1} = (NOPAT_n + DA_n - CAPEX_n - INWC_n) \cdot (1+g) \quad (9)$$

Where g together with r define a reinvestment ratio b that remains constant for t ≥ n, as indicated by formula (7). Let us illustrate with the following example.

The financial projections for a firm show the following results for the last year of the horizon ($t = n$):

EBITn	1300
DAn	250
Tn	100
CAPEXn	280
INWCn	500
FCF_n	**670**

Table 1

Assume that it is expected to maintain a growth rate of 10%. Imagine that the projection for the free-cash flow corresponding to the first year of the perpetuity is computed as follows:

$$FCF_{n+1} = FCF_n \cdot (1+g) = 670 \cdot 1.1 = 737$$

This projection is wrong. In order to perform a consistent calculation we need to know first what is the expected return on reinvested assets (*r*) and how large the investment in net working capital ($INWC_n$) must be to attain such growth rate.

Say that experience suggests a *INWC/EBIT* ratio of 46.15% and that r = 12%. This means that:

$$INWC_n = EBIT_n \cdot 0.4615 = 1300 \cdot 0.4615 = 600$$

and

$$b = \frac{g}{r} = \frac{10\%}{12\%} = 0.833$$

Thus, according to formula (8),

$$CAPEX_n = b \cdot NOPAT_n + DA_n - 0.833 \cdot 1200 + 250 - 600 = 650$$

In consequence, the cash flow projection for $t = n$ must be modified as follows,

$EBIT_n$	1300
DA_n	250
T_n	100
$CAPEX_n$	650
$INWC_n$	600
FCF_n	**200**

Table 2

The free-cash flow at $t = n$ is now 200. Applying the 10% growth rate (that is now consistent with the assumptions on reinvested assets), the right value for the free-cash flow for the first year of the perpetuity will be,

$$FCF_{n+1} = FCF_n \cdot (1+g) = 200 \cdot 1.1 = 200,$$

instead of the previous wrong result of $FCFn_{+1} = 737$.

The potential error of the erroneous approach is enormous. Under the wrong procedure the present value of the perpetuity at t = n would be 3.35 times larger originating a gross overvaluation of the firm.

Ignoring Growth through External Funding

Notice that equation (6) carries an important assumption: growth is possible only through reinvestment of the company´s free-cash flows. That is, earnings plus any financing obtained by new debt, keeping a constant leverage ratio[8]. This signifies that "exogenous

8. For a detailed analysis of the effect of leverage on growth refer to Sabal 2007.

growth", meaning the portion of growth financed through equity and bond issues beyond these bounds, is discarded.

Think of the firm of the previous example having investment opportunities exceeding its current capacity for endogenous growth. The net present value of these ventures can be materialized only if (besides taken advantage of retained earnings and its associated incremental debt) the firm has the possibility of issuing additional new equity and debt. This added value might be very considerable but is ignored when exogenous growth is ignored resulting in this case in a potential undervaluation of the firm.

ERROR 3: When using perpetuities for the computation of the terminal value, the analyst ignores that additional growth could be financed through new equity and debt issues. Thus valuation results might be underestimated.

Impact of Exchange Rates on the Terminal Value

When the valuation is performed in a base currency (say the Euro) that is different from the local currency (say the Turkish Lira) the cash flows are projected in the local currency and then converted into the base currency at a series of estimated exchange rates.

This affects the first free-cash flow of the perpetuity (FCF_{n+1}) since it must also be translated from the local to the base currency at a specified exchange rate (XR_{n+1}).

Recall that by definition the assumptions for the perpetuity remain unchanged. Therefore, it is very important for the exchange rate for conversion (XR_{n+1}) to be sustainable in the long-term. Namely, it must be an exchange rate that is in equilibrium with prices (purchasing power parity) in local and base currency for the first year of the perpetuity ($t = n+1$)[9]. It is possible to maintain over or undervalued exchange rates for a limited period of time but it is not reasonable to translate this disequilibria permanently to the perpetuity.

Let us illustrate with an example. Imagine that the expected free-cash flow at the first year of the perpetuity (FCF_{n+1}) is 1000 LC (local currency) and its exchange rate is expected to be 10 LC/BC (local currency in terms of base currency). However, assume that this exchange rate is not in equilibrium and the equilibrium exchange rate is only 8 LC/BC. This means that LC is undervalued with respect to BC.

If the expected exchange rate is applied to the cash flow, its value in terms of BC will be $\frac{1000}{10} = 1000 BC$

But the equilibrium value should be instead $\frac{1000}{8} = 125 BC$.

Ignoring this adjustment implies an underestimation of the terminal value of $1 - \frac{100}{125} = 20\%$.

Although it remains a challenge to estimate exchange rates that are consistent with purchasing power parity, an effort must be made to avoid grossly misestimating the exchange rate for conversion since this might substantially distort the results due the considerable weight terminal values generally have in the total value.

ERROR 4: Converting the first free-cash flow of the perpetuity at an exchange rate that is inconsistent with purchasing power parity.

Determinants of Long-Term Growth

Value is created only if the net present value of reinvested funds is positive. In other words, the return on reinvested assets r must be larger than the minimum return on assets demanded by investors rA,

$$r > r_A \qquad (10)$$

As a firm matures the return on reinvested assets r is expected to decline gradually until it becomes equal to r_A and no more value is added. At this stage the value of the firm as a going concern is not longer bigger than the market value of its assets, and (theoretically) liquidation is advised.

Using the formula for the present value of a growing perpetuity and allowing for the reinvestment of a fraction (b) of NOPATn+1, the terminal value for the limit case in which r equals rA becomes:

$$TV_n = \frac{(1-b)\cdot NOPAT_{n+1}}{r_A - g} = \frac{(1-b)\cdot NOPAT_{n+1}}{r_A - b\cdot r_A} = \frac{NOPAT_{n+1}}{r_A} \qquad (11)$$

which is equivalent to assuming no growth after the horizon[10].

Naturally, value also ceases to be created, and indeed is destroyed, when the return on reinvested assets r is less than the minimum expected return on assets

9. In practice the equilibrium exchange rate might be difficult to estimate. Its accuracy depends on the historical stability of the local currency vis-à-vis the base (strong) currency and the length of time of the sample employed to determine it.

10. This discussion focus on real returns. The equivalent expression in presence of inflation is $TV_n = \frac{NOPAT_{n+1}}{r_A - \pi}$ (π = inflation rate). For a detailed analysis on this matter refer to Bradley & Jarrell 2008 and 2011, and Friedl & Schwetzler (2011).

r_A. But this means that the firm must have been liquidated before reaching this regrettable situation, that is, when r became equal to rA. Thus, this eventuality is discarded as a suboptimal course of action.

A word of caution is advised at this point. The previous reasoning about liquidation is somewhat divorced from reality. The decision to liquidate is much more complex in practice since it must take into consideration other factors such as: optimum timing of the liquidation process, transaction costs and taxes related to liquidation and, most importantly, the option value of deferring the decision to liquidate. This option means that even though no value might be being added by management, the decision to liquidate must be postponed whenever there is a significant likelihood of improved prospects for the company in the future. With the purpose of centering the discussion on the main issues these considerations are ignored.

Following the previous line of thinking about decreasing returns, some analysts recommend extending the horizon until the point when it is expected for the firm to stop creating value. At this moment a no-growth perpetuity (implying a return on reinvested assets r equal to the minimum return on assets r_A) is specified.

As explained above, this procedure is equivalent to assume liquidation at the end of the horizon and thus demands a projection period as long as the expected remaining life of the firm. This is reasonable as long as free-cash flows can be estimated for such a length of time. But when the expected life of the company is considerable it might make more sense to set up one or more phases of declining growth between the end of the time period during which free-cash flows are projected and the beginning of the no-growth/liquidation stage.

ERROR 5: Assuming a no-growth perpetuity before the growth phase of the firm is exhausted, simply because it is hard to project for a longer period of time.

Return on Reinvested Assets

The extent during which a company is expected to create value, keep r > r_A, and justify its existence depends on many factors. The most relevant are: the length of the product cycle, persistence of competitive advantages and, potential for product renewal (Koller, Goedhart & Wessels 2010).

The length of the product cycle depends on the type of product. For instance, a model of passenger jet generally has a longer life cycle than a particular perfume. Competitive advantages are kept longer the more difficult the know-how sustaining them is to imitate. For example a patented medicine usually has a longer life than a (non-patented) financial product. The potential for product renewal is associated with the dynamism of the firm. Some successful companies such as IBM and Apple keep reinventing themselves extending their life span beyond that of many of their original competitors.

ERROR 6: Assuming a return on reinvested assets with little or no consideration to the characteristics of the industry and the firm.

Growth Rate Bounds

A growth rate larger than long-term economic growth for infinity implies in the end a firm bigger than the world economy. This absurdity places an upper-bound to the growth rate.

But what is the significance of smaller growth rates? Thinking now in nominal terms let us focus on three key cases:

a. Nominal Growth Rate equals Zero (no-growing perpetuity): With positive inflation this means gradual decline in real terms.

b. Nominal Growth Rate equals Inflation: This implies no real growth. In a growing market this means a declining market share.

c. Nominal Growth Rate equals Market Growth: The firm grows at the same rate than the market where it operates meaning that market share is maintained forever. Naturally, market growth must always be lower than long-term economic growth. If not, several growth stages must be set up before reaching a perpetuity.

Another obvious restriction is that the nominal growth rate must always be smaller than the discount rate, otherwise, the present value of the perpetuity will tend to infinity as growth approaches the discount rate or turn negative when growth surpases the discount rate, both of them meaningless outcomes.

Naturally, all growth rates might be realistic and consistent with the characteristics of the company being valued.

ERROR 7: The nominal growth rate is supposed larger than long-term economic growth, or specified without due consideration to inflation, market growth or market share.

As a final comment observe that long-term growth rates are upper-bounded. But the return on reinvested assets is not necessarily so as long as some competitive advantage persists.

Only the possibility to keep investing at high rates is unsustainable in the long-term (that is, b must start decreasing at some point).

Conclusions

The valuation of firms with the discounted cash flow method demands the determination of a projection period, or horizon. This poses the problem of valuing the free-cash flows occurring after the horizon. These free-cash flows are usually represented by a single cash flow at the end of the horizon, the so called Terminal Value.

The most popular methods of quantifying terminal value are: a) assuming that the company will cease to exist and estimating a liquidation value; b) calculating a multiple based on comparable firms and; c) assuming that the company will operate "forever" and thus calculating the final value as the present value of a perpetuity. In this paper attention was centered on the terminal value expressed as the present value of a perpetuity.

It has been argued that important mistakes are often made when working with terminal value perpetuities in company valuations. Seven important errors have been identified. They are mainly caused by the following misjudgments:

- **a.** Failure to correctly account for the instability of the free-cash flows during the projection horizon.
- **b.** Lack of consistency between the last free-cash flow of the horizon and the assumptions about reinvestment and growth.
- **c.** Failure to account for the possibility to grow through external funding.
- **d.** Inconsistency between the market exchange rate and the equilibrium exchange rate when working in different currencies.
- **e.** Ignoring the possibility of different phases of growth.
- **f.** Setting up growth rates with little concern for the characteristics of the firm and the industry, inflation, market growth or market share.

It is expected that this paper will help practitioners to avoid discrepancies when dealing with terminal value perpetuities. This surely will result in a more reasonable range of possible values for the firm.

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